Touring the Galilean Satellites

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A technique is developed for exploring orbiting Jupiter spacecraft to repeatedly encounter the Galilean satellites. Commensurable orbits are derived, assuming coplanar two-body motion, which mesh with the motion of Io, Europa, and Ganymede to provide multiple satellite flybys. A practical 14-day orbit is presented for the 1981 Jupiter opportunity which provides 35 satellite encounters, including several Callisto flybys, in a period of 170 days. Encounter orientations are favorable for surface imagery and radio occultation. Satellite gravitational perturbations, however, are sufficient to upset the predicted encounter sequences. An inexpensive impulse policy is presented which provides the needed orbit control.

Introduction

A NUMBER of planetary satellites are of considerable scientific interest in their own right. The four Galilean satellites of Jupiter, namely Io, Europa, Ganymede, and Callisto, are among them. These bodies form the basis of a mini "solar system" of 12 satellites which orbit Jupiter. Of the four, Ganymede is larger than the planet Mercury, and Io and Callisto are both larger than the Earth's moon. A spacecraft operating in the vicinity of the satellites could collect data essential to a better understanding of the origin and evolution of the Jovian system and these bodies. Surface imagery, infrared radiometry, precision tracking, and radio occultation are typical spacecraft measurements which could unveil some of the major events of the satellites' histories.

This article develops a method for using an orbiting Jupiter spacecraft to repeatedly observe the Galilean satellites in a flyby mode. The idea was conceived within the concept of a Jupiter orbiter mission[1] and the requirements appear to be consistant with the larger goal of Jupiter exploration. A key feature of the technique is the characteristic of repetitive satellite encounters which result from matching the spacecraft's orbit with the resonant motion of Io, Europa, and Ganymede.

Satellite Motion

The orbits of the four Galilean satellites of Jupiter are depicted in Fig. 1. The analysis assumes that all four satellites move in circular equatorial orbits† about Jupiter at mean angular rates. The orbit radii are set equal to the semimajor axes[2] a, given in Fig. 1, in units of Jupiter's equatorial radius (71,372 km). The mean longitudes (in degrees) of the satellites at any epoch t, as taken from Sampson,[3] are

$$L_I = 142.59987 + 203.48895(t - 2415020.0) \quad (1a)$$

$$L_E = 99.55081 + 101.37472(t - 2415020.0) \quad (1b)$$

$$L_G = 168.02628 + 50.31761(t - 2415020.0) \quad (1c)$$

$$L_C = 234.40790 + 21.51707(t - 2415020.0) \quad (1d)$$

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† Callisto has the largest eccentricity, $e = 0.0075$, and all four satellites' inclinations are given as $\sim 0°$ to the Jupiter equator.[2]

where t is expressed in Julian days, L is measured relative to the Earth's equinox and ecliptic of 1900.0 (Julian date 2415020.0), and the subscripts denote the respective satellites. For the purpose of determining satellite positions the small angles of Jupiter's orbit inclination and equatorial obliquity to the Earth's ecliptic are ignored.

Laplace is generally credited for discovering the orbital resonance between Io, Europa, and Ganymede which, expressed in terms of mean longitudes, is

$$L_I - 3L_E + 2L_G \cong \pi \quad (2)$$

This relationship results from the fact that the orbital periods of Io, Europa, and Ganymede are very close to the ratio 1:2:4, respectively. In addition to Eq. (2), it is possible to find an epoch when

$$L_I = L_G = L_E + \pi \quad (3)$$

which is rigorously satisfied. This situation, often referred to as syzygy, is also shown in Fig. 1. It repeats itself about every 7 days (Ganymede's period). Because the orbit periods of the first three satellites are not exactly in the ratio 1:2:4, the line of syzygy precesses in a retrograde sense relative to the Jupiter-sun line with a period of 437 days. The motion of Callisto, the fourth Galilean satellite, is not commensurate with the system of Io, Europa, and Ganymede, but does contribute secular perturbations which are important when determining the actual orbits of all four satellites.

The principle whereby Jupiter orbits exhibit repetitive encounters with the first three Galiliean satellites can be stated as follows: If, in fact, the orbit periods of Io, Europa, and Ganymede are exactly in the ratio 1:2:4, and if an elliptical

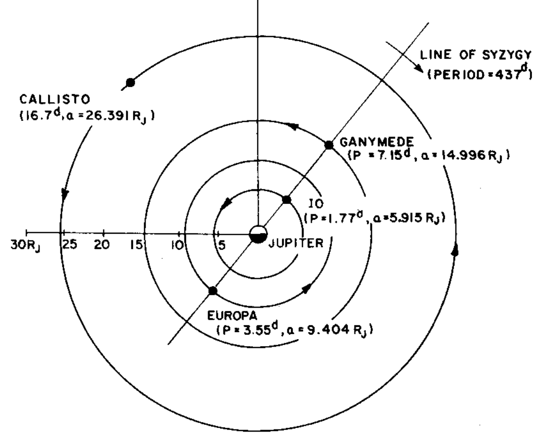


Fig. 1 Galilean satellites of Jupiter.

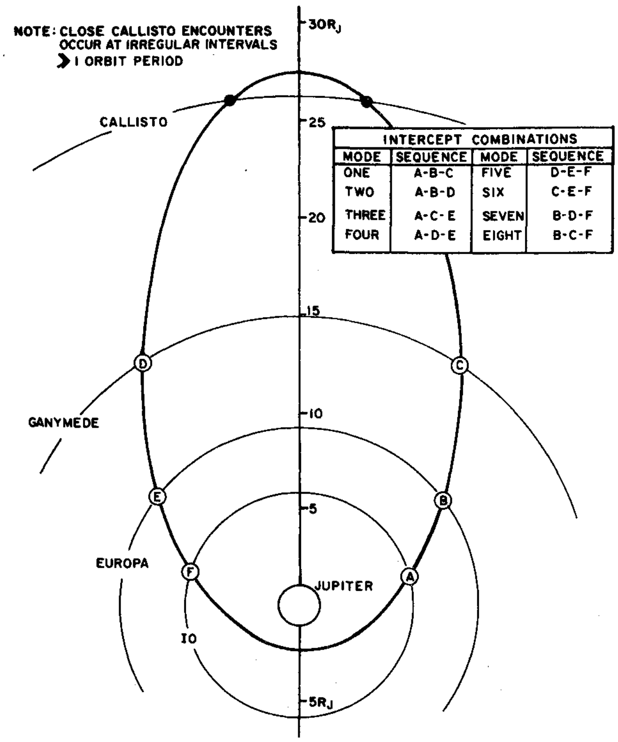


Fig. 2 Orbit configuration and intercept combinations.

Jupiter orbit with a period equal to an integer number of any of these periods can be established such that a spacecraft intercepts all three satellites on one orbit, then it will intercept these satellites on every orbit.

The existance of this principle, 1) depends upon the assumption of an exact ratio between the satellite periods, 2) requires coplanar orbits, and 3) neglects any orbit perturbations from the satellites' masses. All three factors are assumed here in developing a method for determining the elliptical orbits of interest. Nominal orbits and their encounter characteristics, discussed below, are based on actual satellite periods rather than an exact ratio of 1:2:4, but still require the constraints of coplanar orbits and massless satellites. The effects of satellite perturbations are treated after the nominal encounter orbits have been defined.

The first step in determining proper orbits for repetitive satellite encounters is to identify positions of intercept. Using simple two-body relations, the spacecraft's elliptical orbit crosses a circular satellite orbit when its true anomaly, η, satisfies the equation

$$\eta = \cos^{-1}[1/e(p/R_S - 1)] \tag{4}$$

where e and p are the eccentricity and semilatus rectum of the spacecraft's orbit, respectively, and R_S is the circular orbit radius of the satellite. The first such orbit crossed by the spacecraft after periapsis would be that of the satellite Io. If the initial orbital position of the spacecraft is adjusted such that an encounter with Io occurs at the time of orbit crossing, t_0, then the mean longitude of Io at t_0, measured from the spacecraft's orbit periapse point, must be

$$\lambda_I(t_0) = \cos^{-1}[1/e(p/R_I - 1)] \tag{5}$$

At this same time, t_0, the positions of Europa and Ganymede, in the same planar reference system are denoted as $\lambda_E(t_0)$ and $\lambda_G(t_0)$, respectively.

With the Io intercept thus established at t_0, the spacecraft is permitted to continue its unperturbed path about Jupiter. In order for the spacecraft to intercept the next satellite, Europa, when its orbit is crossed at t_1, Europa's new position $\lambda_E(t_1)$, must satisfy the equation

$$\lambda_E(t_1) = \cos^{-1}[1/e(p/R_E - 1)] \tag{6}$$

But Europa's position must also satisfy the relationship

$$\lambda_E(t_1) = \lambda_E(t_0) + \Delta\lambda_E(t_1,t_0) \tag{7}$$

where Europa's travel angle, $\Delta\lambda_E$, since t_0 is just

$$\Delta\lambda_E(t_1,t_0) = n_E(t_1 - t_0) \tag{8}$$

if n_E is the mean motion of Europa in its circular orbit. Note also that the times t_0 and t_1 are related to the spacecraft's respective orbital positions by the expressions

$$t_0 = [E(t_0) - e\sin E(t_0)]/n_{sc} \tag{9a}$$

$$t_1 = [E(t_1) - e\sin E(t_1)]/n_{sc} \tag{9b}$$

where $E(t)$ is the eccentric anomaly, n_{sc} is the spacecraft's mean orbital motion and the time of periapse, t_p, is taken as zero. By combining Eqs. (7), (8), and (9) with (6) the initial position of Europa, $\lambda_E(t_0)$, which insures an intercept at t_1 is determined. The resulting expression for $\lambda_E(t_0)$ is

$$\lambda_E(t_0) = \cos^{-1}[1/e(p/R_E - 1)] - n_E/n_{sc}\{E(t_1) - E(t_0) - e[\sin E(t_1) - \sin E(t_0)]\} \tag{10}$$

The same line of reasoning would lead to a similar solution for the proper initial position of Ganymede, $\lambda_G(t_0)$, to insure an intercept between it and the spacecraft at its orbit crossing time, t_2.

The four orbital parameters e, p, E, and n_{sc} appearing on the right-hand side of Eq. (10) can be re-expressed in terms of the spacecraft's orbit periapse radius, R_p, and orbit semimajor axis, a, as

$$e = (a - R_p)/a \tag{11a}$$

$$p = R_p(2a - R_p)/a \tag{11b}$$

$$E = \cos^{-1}[(a - R_S)/(a - R_p)] \tag{11c}$$

$$n_{sc} = [\mu_J/a^3]^{1/2} \tag{11d}$$

where μ_J is the gravitational constant of Jupiter.

The general expression for the initial position of any coplanar satellite, $\lambda_S(t_0)$, which provides an intercept at the time of orbit crossing is formed by substituting Eqs. (11) into Eq. (10). The result is

$$\lambda_S(t_0) = \cos^{-1}\{a/(a - R_p)[R_p/R_S(2 - R_p/a) - 1]\} - (a^3/\mu_J)^{1/2}n_S\{f[R_S] - f[R(t_0)] - (1 - R_p/a)[\sin(f[R_S]) - \sin(f[R(t_0)])]\} \tag{12}$$

where $f[R] = \cos^{-1}[(a - R)/(a - R_p)]$. For the first intercept, which establishes the reference time, t_0, R_S equals $R(t_0)$ leaving only the first term in Eq. (12).

The only independent variables of Eq. (12) are R_p and a. The semimajor axis, a, may be re-expressed as a function of the spacecraft's orbit period, P, as

$$a = (\mu_J P^2/4\pi^2)^{1/3} \tag{13}$$

The characteristic of repetitive observations requires that P be an integer value of Io's period P_I, as are those of Europa and Ganymede. Therefore, a is constrained to take on only the values

$$a = (\mu_J k^2 P_I^2/4\pi^2)^{1/3},\ k = 1,2,4,8,12,16,\ \ldots \tag{14}$$

Furthermore, if Io is to be intercepted, the remaining free parameter, R_p, must be in the interval $R_J < R_p < R_I$ where R_J is the equatorial radius of Jupiter.

Equation (2) is now reintroduced as an equality

$$\lambda_I - 3\lambda_E + 2\lambda_G = \pi \tag{15}$$

Table 1 Multi-satellite-intercept orbits

	Orbit period, days			
Orbit parameters	3.556 (k = 2)	7.111 (k = 4)	14.222 (k = 8)	21.333 (k = 16)
Mode 3 (or 7):				
Periapse radius, R_J	2.556	2.391	2.290	2.255
Apoapse radius, R_J	16.260	27.483	45.131	59.884
Orbit capture impulse,[a] km/sec	3.324	2.250	1.624	1.384
Orbit radiation lifetime[b]				
revolutions	182	135	109	101
days	648	957	1548	2149
Intercept longitudes, deg				
Io, $\lambda_I(t_0)$	110.418	107.877	107.035	106.717
Europa, $\lambda_E(t_2)$	223.277	230.658	233.548	234.535
Ganymede, $\lambda_G(t_1)$	165.218	147.132	141.618	139.863
Mode 4 (or 8):				
Periapse radius, R_J	1.368	1.250	1.213	1.201
Apoapse radius, R_J	17.451	28.624	46.208	60.938
Orbit capture impulse,[a] km/sec	2.400	1.618	1.180	1.009
Orbit radiation lifetime[b]				
revolutions	4.9	3.1	2.7	2.6
days	17.5	22.2	38.6	55.3
Intercept longitudes, deg				
Io, $\lambda_I(t_0)$	132.012	130.566	129.317	128.818
Europa, $\lambda_E(t_2)$	211.283	215.560	217.894	218.757
Ganymede, $\lambda_G(t_1)$	193.745	203.607	207.486	208.817

[a] Capture impulse is based on a typical hyperbolic approach speed of 7.25 km/sec for Jupiter orbiter missions.
[b] Lifetime is based on a trapped-radiation model developed in Ref. 1.

Then for a given value of k, the three equations represented by Eq. (12), along with Eq. (15), provide four equations for the four unknowns λ_I, λ_E, λ_G and R_p. The solution of this set is, perhaps, most easily accomplished by iterating on the orbit periapse, R_p. A value of R_p is picked within the permitted interval and the values of λ_I, λ_E, and λ_G are computed from Eq. (12). A remainder, F, is then computed from Eq. (15), i.e.,

$$F = \lambda_E - 3\lambda_E + 2\lambda_G - \pi \quad (16)$$

This sequence is repeated for new values of R_p until F is reduced to zero, providing the correct orbit solution.

Orbits

The orbital geometry of satellite intercepts is depicted in Fig. 2. There are, in theory, eight possible sequences, or Modes, whereby a Jupiter orbiting spacecraft could intercept Io, Europa, and Ganymede in one elliptical orbit revolution. These Modes are listed in the table included in Fig. 2. Orbit solutions for Modes 5-8 are identical to those for Modes 1-4, respectively, their intercept sequences being symmetrical to the first four relative to the orbit apseline.

Four Mode 3 (or 7) and Mode 4 (or 8) orbit solutions are presented in Table 1 for k = 2, 4, 8 and 16 along with relevant orbit data. The orbit periods given were not derived from the exact ratio assumption of periods of Io, Europa, and Ganymede. Rather, they have as a base period an average of the actual periods of the first three satellites normalized to Io's period. No solutions were found for Mode 1 (or 5) and Mode 2 (or 6) with periapse radii larger than the equatorial radius of Jupiter (71,372 km).

It is intuitively preferrable to select for application the shortest period orbit possible since it will provide the most encounters over a fixed length of time. However, in addition to the orbit period three other mission related parameters cited in Table 1 should be considered: 1) the required capture impulse with typical Jupiter approach conditions, 2) the orbit radiation lifetime and 3) whether or not Callisto encounters are possible. High capture impulse, short radiation lifetime and no chance for Callisto encounters ($R_a < R_C$) combine to make the 3.556-day orbits for both Modes 3 and 4 unattractive. The short lifetimes of the remaining three orbits cited for Mode 4 (or 8) distracted from their utility, although the lower orbit periapse radii do reduce the required capture impulse compared to respective period orbits for Mode 3 (or 7). From the three remaining orbit possibilities of Mode 3, the 14.222-day orbit is, perhaps, the best compromise between capture impulse, radiation lifetime, and potential number of satellite encounters.

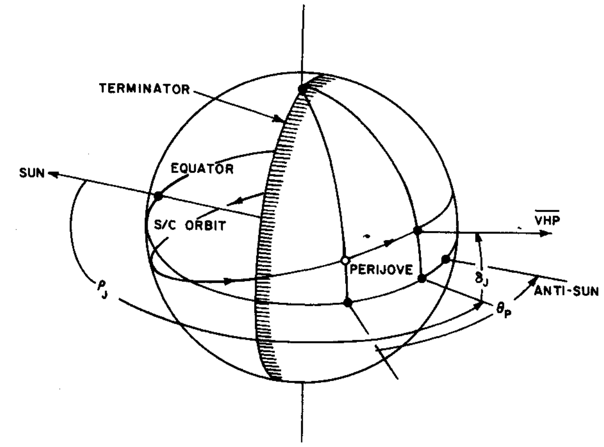


Fig. 3 Spacecraft orbit orientation at Jupiter.

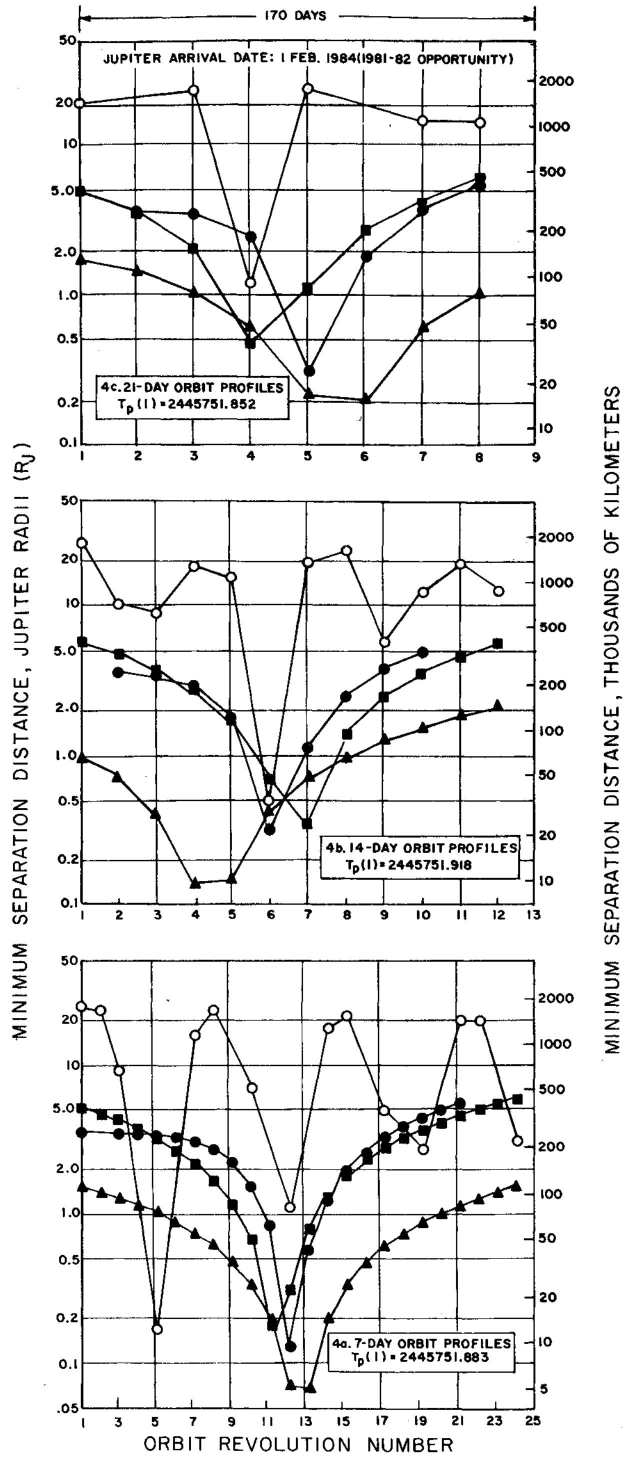
Encounter Histories

From Fig. 1 it was shown that the periods of Io, Europa, and Ganymede are not in the exact ratio 1:2:4 causing the line of syzygy of the three satellites to rotate in a retrograde sense relative to the Jupiter-sun line with a period of 437 days. On the other hand, the spacecraft orbits presented in Table 1 are relatively stable.‡ Therefore, the correct orbital phasing between spacecraft and satellites only exists at specific times. By generating real-time examples of encounter histories not only is it possible to verify that the computed orbits do, in fact, exhibit repetitive satellite encounters, but also the favorable phasing times for particular orbit/mode combinations can be identified, and the encounter characteristics immediately before and after the correct phasing time can be investigated.

It is necessary, before generating encounter histories, to locate the orbit periapse in Jupiter's equatorial plane (coplanar with the Galilean satellite orbits) at some given epoch. The proper orbit orientation is that which corresponds to a periapse capture maneuver on the arrival date of an Earth-Jupiter interplanetary transfer. The orbital positions, arrival dates and related approach conditions are given in Table 2 for two Jupiter orbiter mission opportunities, 1981-82 and 1983, which were used to generate the satellite encounter characteristics presented below. The angles cited in Table 2 are defined by the diagram presented in Fig. 3. The arrival conditions correspond to a 760-day Earth-Jupiter trajectory which is typical of transfers yielding maximum orbit payload across the 12-yr Jupiter opportunity cycle. The declination of the approach asymptote, δ_J, to Jupiter's equator is taken as zero in this analysis for consistancy with the assumption of coplanar orbits. Since δ_J generally has a nonzero value, a Jupiter orbit plane change of equal or greater magnitude would be required in an actual mission application to achieve an orbit with the multiple satellite encounter characteristics described here. The associated impulse penalty can be as high as 350 m/sec for a bad opportunity, e.g., 1984, and less than 25 m/sec for a favorable opportunity, e.g., 1980-81.

Now consider as a first example of encounter histories the 7-day orbit for Mode 3. Assuming the orbit is established from a 760-day transfer of the 1980-81 Jupiter opportunity, the resulting encounter histories with all four Galilean satel-

‡ Orbit precession is dominated by Jupiter's oblateness, which for the Mode 3 14-day orbit produces an orbit regression rate which is 14 times slower than that of the satellites' line of syzygy. For encounter history analysis, spacecraft orbits were, therefore, assumed inertially fixed.



SATELLITE	SYMBOL	RADIUS(KM)	SOI*(R_J)	LDSSD†(R_J)
IO	●	1800	0.101	5.17
EUROPA	▲	1550	0.135	4.52
GANYMEDE	■	2620	0.347	7.89
CALLISTO	○	2390	0.504	7.31

*SOI: SPHERE OF INFLUENCE
†LDSSD: LUNAR DISC SIZE SEPARATION DISTANCE

Fig. 4 Mode 3 satellite encounter histories.

lites are presented in Fig. 4a. The minimum separation distance (in Jupiter radii along the left ordinate and in kilometers along the right ordinate) with each satellite on each orbit is plotted for a period of 24 orbits, or 170 days. The center of this time interval occurs approximately where Io, Europa, and Ganymede have their closest encounters. The encounters with these first three Galilean satellites decrease to a minimum value at or near the computed alignment epoch and then increase again as the satellites' line of syzygy continues to regress about Jupiter. The encounter profile with Callisto is somewhat less regular but systematic, and includes several encounters within 10^5 km of the satellite. When the minimum encounter distance exceeds the radius of the satellite's orbit it is no longer plotted (an arbitrary decision for presenting the data).

The sequence shown in Fig. 4a commences 20 days after arrival (2/1/84) at Jupiter. Over an interval of 170 days there are 73 encounters for which the apparent satellite diameters are larger than that of the moon as seen from Earth. That's an average of one encounter every 2.3 days. Encounter histories for the 14-day and 21-day orbits of Mode 3 (same launch opportunity) are presented in Figs. 4b and 4c, respectively. The total time interval (170 days) for these profiles is the same as that for the 7-day orbit example. What becomes immediately obvious from Fig. 4 is that the number of encounters within a fixed period of time is inversely proportional to the orbit period. The 14-day orbit has half as many encounters as the 7-day orbit, while the 21-day orbit has only one-third as many. Closer examination also reveals that the number of close Callisto encounters also decreases.

It is possible to control, to some extent, the minimum encounter distance from a satellite and the orbit on which it occurs by varying the orbit periapse time, t_p. In Fig. 4 t_p has been adjusted to raise the minimum encounter distance above each satellite's sphere-of-influence, SOI, whenever possible. This was done to strengthen the assumption of no satellite perturbations (massless satellites), and to illustrate the available control over the encounter history profiles. It should be noted, however, that for the 7-day orbit the relative change in the Europa and Ganymede encounter distances between adjacent orbits is too small to keep all encounters outside the respective SOI's. The two closests Europa encounters have been equalized in order to make them as large as possible. All minimum encounters for the 14-day and 21-day orbits are greater than all respective SOI's. It was suggested earlier that, from the standpoint of mission application, the 14-day orbit represents the best compromise between operational constraints and number of encounters. The orbit perturbations likely to result from the close Europa and Ganymede encounters for 7-day orbits add further support to this suggestion.

During one syzygistic period (486 days with respect to the fixed stars) an opportunity for multiple satellite encounters will occur for each encounter mode which has a realistic orbit solution, i.e., Modes 3, 4, 7, and 8. Encounter opportunities for 14-day orbits are charted in Fig. 5 in the order in which they occur: Modes 3, 4, 8, 7, and back to 3 again. Bear in mind, however, that although Modes 4 and 8 have been included in Fig. 5 for completeness, the associated radiation lifetime predictions (Table 1) for their orbits are far too small to make them practical.

Table 2 Jupiter approach and orbit conditions

Parameters		Opportunities 1981–82	Opportunities 1983
Transfer time, days		760	760
Arrival date		Feb. 1, 1984	Feb. 27, 1985
Asymptotic approach conditions			
Speed, km/sec		7.26	7.12
ρ_J, deg		129.8	125.4
δ_J, deg		3.12	5.99
Orbit periapse angle, θ_p, deg			
Mode 3 (or 7):	7-day orbit	71.18	75.20
	14-day orbit	70.75	74.78
	21-day orbit	70.68	74.63
Mode 4 (or 8):	7-day orbit	65.57	69.69
	14-day orbit	65.35	69.47
	21-day orbit	65.28	69.40

As mentioned previously, and as can be seen from Fig. 5, Mode 3 begins about 20 days after the 1980–81 opportunity arrival date, Mode 4 begins 108 days after the same arrival, Mode 8 begins 256 days after arrival, and Mode 7 begins 366 days or one year after arrival. By the time the satellites' line of syzygy is again properly aligned for Mode 3, the 1983 opportunity arrival date (Feb. 27, 1985) has passed so that the spacecraft's orbit can be reoriented to match 1983 transfer approach conditions (see Table 2). As a result the second Mode 3 encounter sequence commences 79 days after the arrival date of the 1983 opportunity or about 35 days sooner than it would have with an inertial orbit from the 1981 opportunity.

Encounter Geometry

The utility of near satellite encounters is closely related to the direction of the sun (and Earth) during the flyby. Several consecutive flyby paths for Ganymede encounters§ are plotted in Fig. 6 for the 14-day orbit Mode 3 example discussed previously. Relative flyby paths (numbered to correspond with orbits in Fig. 4b) are presented in a satellite reference frame with the sun toward the top of the figure. The variable direction of the Earth during the 170-day encounter sequence is also shown. Perturbations from Ganymede have been ignored in the computation of the flyby paths. The spacecraft approaches the satellite from the general direction of the sun toward the day side of Ganymede. This configuration makes it possible for the spacecraft to locate the satellite well before flyby. Closest approach points occur at or near Ganymede's terminator providing good surface shadow detail (assuming there is surface relief) for visual imagery. Improved resolution IR measurements can also be performed across the terminator. These favorable lighting conditions are retained on consecutive encounters which are laterally displaced but have the same relative orientation to the sun. One hour time intervals measured from the closest approach point have been added to the nearest encounter path (#7). It is apparent that adequate time is available to perform surface imagery under varying conditions of solar illumination and resolution. Finally, it can be observed from Fig. 6 that radio occultations are possible during the early encounters with at least one occultation occuring when the spacecraft is less than one Jupiter radius from Ganymede.

These encounter characteristics for Ganymede apply to encounters with all four Galilean satellites with one exception for Callisto. Since Callisto is not included in the commensurable satellite motion, close encounters occur in a more or less random fashion. Hence, laterally displaced consecutive orbit encounters do not apply for Callisto. However, the favorable flyby orientation previously described still holds when close encounters occur. This orientation and the related encounter characteristics can be traced to the Jupiter orbit capture point of typical orbiter approach conditions (see Table 2). Since these approach conditions are quite stable for all Jupiter mission opportunities, it can be concluded that generally favorable satellite flyby conditions will prevail.

Orbit Control

The encounter sequences of the satellite-synchronized orbits predicted by the two-body analysis presented previously can most easily be upset by two factors: 1) uncorrected errors of the orbit capture maneuver and 2) satellite gravitational perturbations acting on an initially correct orbit. The effects of errors in initial orbit conditions are discussed first. The Ganymede encounter profile of the 14-day orbit Mode 3 example presented in Fig. 4b will be used for the purpose of illustration. The nominal Ganymede encounter sequence is shown in Fig. 7 as a solid curve. Errors in four parameters of the capture maneuver were studied which directly alter the achieved orbit. These parameters are 1) periapse altitude, 2) orbit apseline position, 3) capture impulse time, and 4) capture impulse. Reasonable errors¶ in the first three parameters produce minor positional errors in the initial orbit which do not measurably alter the predicted encounter profiles. However, a 10 m/sec error in the magnitude of the capture impulse which results in a 5.2-hr change in orbit period markedly changes the anticipated encounter sequence. The perturbed Ganymede encounters are plotted in Fig. 7 as solid squares. The results are obviously disastrous. The fact that a period change is equivalent to an accumulative error in time quickly destroys any chance for continued repetitive encounters with Ganymede. The same effect is to be expected with the other satellites.

Even if the initial orbit is perfectly established it will be subsequently altered by gravitational perturbations from the satellites themselves. To determine the significance of these perturbations, the 14-day orbit Mode 3 example was numerically integrated in two dimensions through the period of

§ A complete set of figures illustrating the flyby paths with all four satellites for the 14-day orbit Mode 3 example is presented in Ref. 4.

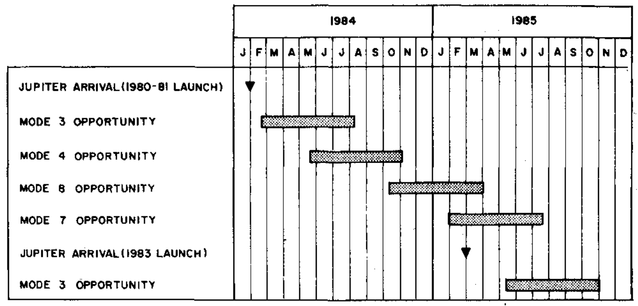


Fig. 5 Galilean satellite tour opportunities.

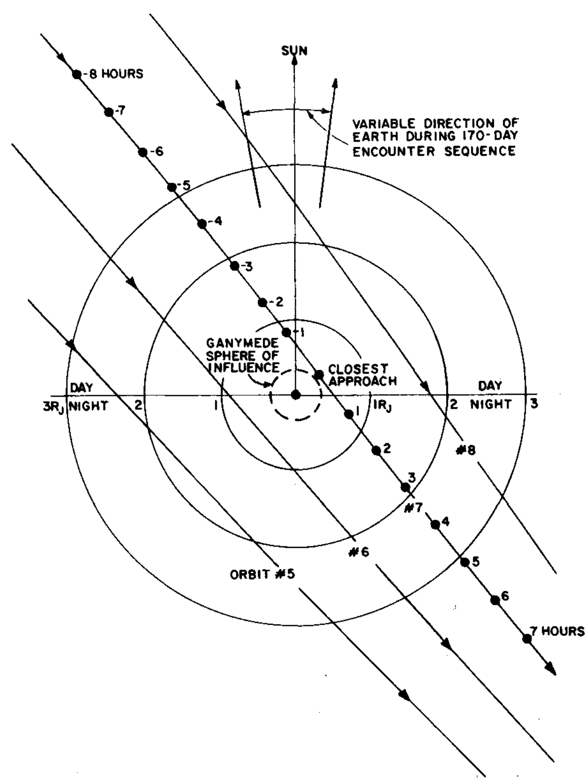


Fig. 6 Close encounter paths at Ganymede, 14-day orbit/mode 3.

¶ A more complete discussion of these errors, including plotted encounter results, is presented in Ref. 4.

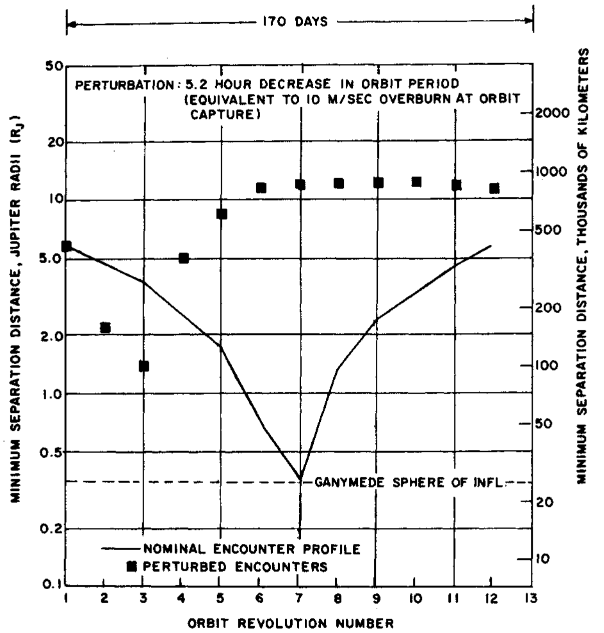


Fig. 7 Orbit period effect on Ganymede encounters, 14-day orbit/mode 3.

encounters presented in Fig. 4 with all four Galilean satellite masses present. Even though none of the satellites' SOI's are penetrated, their perturbations were found to be sufficient to upset the anticipated encounter sequences after several orbit revolutions. The variations in orbit period resulting from these satellite perturbations are presented in Fig. 8a. Recalling the effect of a 5.2-hr (0.217-day) error in period on Ganymede encounters (Fig. 7) it is obvious that these uncontrolled period variations will also destroy the desired synchronization for repeated encounters. Hence, it must be concluded that the encounter sequences presented in Fig. 4 cannot be realized unless some form of active orbit control is employed to cancel the disrupting effect of the satellite gravities.

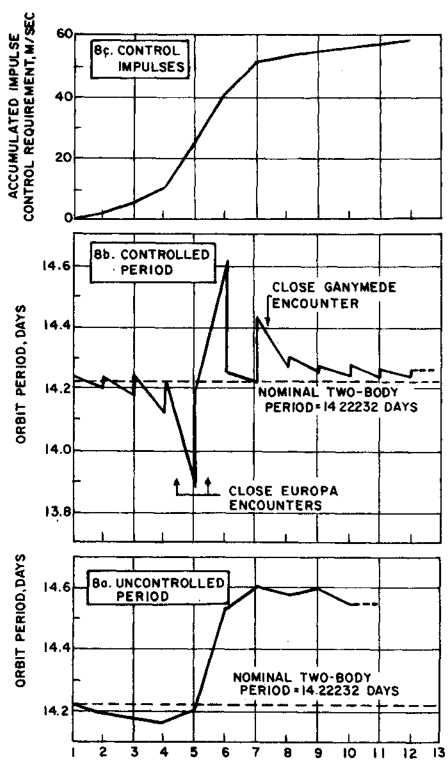


Fig. 8 Satellite perturbation control characteristics, 14-day orbit/mode 3.

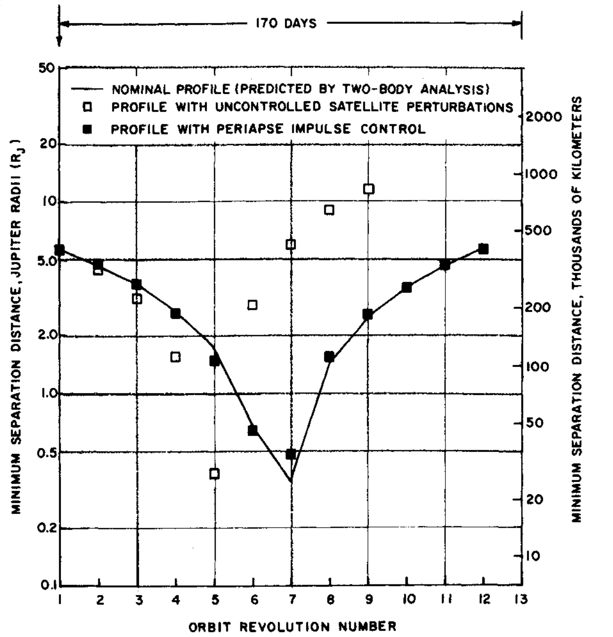


Fig. 9 Encounter profile comparison at Ganymede, 14-day orbit/mode 3.

A relatively simple impulsive orbit control policy may be used to solve this problem. Essentially, achieving the predicted satellite encounters is directly related to achieving a like number of equally spaced orbit periapse passage times which form a reference time line for the encounters. The control policy selected requires that the nominal orbit period be changed at orbit periapse an amount equal and opposite to the anticipated change in the next periapse passage time due to satellite perturbations. The periapse impulse required for a desired change in orbit period is very well approximated for the elliptical orbits of interest by the following expression

$$\Delta V \doteq (R_p V_p / 3R_a P)\Delta P \tag{17}$$

where R_p, V_p, R_a, and P are the periapse radius, periapse velocity, apoapse radius, and two-body period, respectively, of the orbit at the time of the impulse, ΔV. The impulse is executed along the periapse velocity vector, its direction being determined by the sign of the desired period change, ΔP.

This control policy was applied to the 14-day orbit of the Mode 3 example (1980–81 opportunity) with very satisfactory results. The controlled period history is presented in Fig. 8b. The vertical steps in the curve are the period adjustments being made by small impulses at each periapse passage. Period changes between successive periapse points have been approximated by straight lines only for convenience. In reality, the orbit period varies over each orbit in such a way that the average period is equal to a nominal value of 14.222 days.

The accumulated impulsive control requirement is presented in Fig. 8c. The total requirement over 12 orbit revolutions is just under 60 m/sec, which is less than 4% of the 1.624 km/sec cited in Table 1 for the capture impulse of this 14-day orbit. The two largest control impulses of about 15 m/sec occur after the two close Europa encounters on orbit numbers 4 and 5 (see Fig. 4b).

A comparison between the Ganymede encounter profile predicted by the two-body analysis and satellite perturbed encounter profiles, with and without orbit control, is presented in Fig. 9. The controlled encounters are in good agreement with the nominal sequence predicted by two-body analysis. This agreement was also verified for the Io, Europa and Callisto encounter profiles. On the other hand, without control the orbit number 5 Ganymede encounter is the only such encounter to come within 100,000 km for any of the four satellites. With orbit control 13 encounters fall within 100,000 km of the satellites.

There are two difficulties associated with the suggested periapse impulse control policy which bear mentioning. Firstly, the amount of control impulse used during any one periapse passage is determined from the anticipated perturbations during the next orbit revolution. Obviously, the perturbations estimate is directly affected by the accuracy of the satellite ephemerides used to determine the perturbations. The sensitivity of this estimate to such errors has not been assessed.

Secondly, the control maneuver must be performed at or near periapse (for maximum effect) where it is most likely to interfere with science experiment operations of the spacecraft. Most of the science instruments of a Jupiter orbiter which are concerned with the planet itself, e.g., imagers, spectrophotometers, and radiometers, provide their best resolution data near periapse. Added to this is the fact that some of the Io encounters occur near periapse for the more practical Mode 3 and Mode 7 orbits. Solutions to this conflict of spacecraft duties, such as turning off planetology experiments for the entire period of encounters or not performing a control impulse on every orbit, have not been analyzed. Both of these problems should be considered in future more detailed studies of the multiple satellite encounter technique.

Conclusions

It is concluded that practical Jupiter orbits are available which provide multiple observations of all four Galilean satellites. Orbit radiation lifetime and capture impulse requirements suggest the use of 14.222-day orbits together with either the Mode 3 or Mode 7 encounter sequences. This combination typically provides 35 close encounters (maximum satellite disc size larger than the moon as viewed from earth) in a period of 170 days. These encounters occur with a frequency of about one every 5 days. The approach direction is always from the sun, which eases the task of locating the satellite well before closest approach. More than 6 hr during flyby are available for surface imagery at variable conditions of solar illumination and resolution. With proper orbit control radio occultations should be possible during at least one encounter with each satellite within a range of one Jupiter radius (~70,000 km).

Initial orbit conditions must be achieved accurately to follow the predicted encounter profiles. Errors in orbit period are particularly unacceptable. Once the encounters begin it will also be necessary to perform small but essential orbit control impulses to counter the disruptive effects of satellite perturbations on the reference orbit.

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Switching Conditions and a Synthesis Technique for the Singular Saturn Guidance Problem

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A singular optimal guidance problem which was motivated by difficulties encountered in the Saturn V AS-502 flight has been studied. It is shown that if the guidance equations are based upon a singular version of the flat-Earth problem, then the control must be discontinuous at a junction of singular and nonsingular subarcs for almost all cases. A good suboptimal guidance scheme based upon a nonsingular approximation of the singular problem is presented. The resultant suboptimal control is continuous, which is more desirable then a discontinuous control, and causes only a noise-level difference in payload.

Introduction

In the second flight of the Saturn V vehicle (AS-502), two engines shut down early in the S-II stage. The measurements received by the onboard guidance scheme, the Iterative Guidance Mode (IGM),[1] indicated that only one engine was out. This resulted in a steep planar steering program in the S-IVB stage which caused the time rate of change of the steering angle to reach its limiting value for a large portion of the S-IVB flight. Since the IGM is based on unconstrained variational theory the resultant trajectory did not reach the desired terminal orbit.

In the aforementioned flight a large disturbance caused the guidance law to determine a steering angle rate of change which was too large. Thus, it would be desirable to design the guidance logic in such a way that the time rate of change of the steering angle is a bounded control variable, say u with $|u| \leq K$, and such that the steering angle is a state variable since it cannot change rapidly (because of physical and reliability constraints). However, the resultant optimal control problem is a singular problem, and the variational and computational theory for such problems is far from satisfactory.

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